# **Peel Mechanics for an Elastic-Plastic Adherend**

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### **Synopsis**

The force required to propagate a 180° bend in an elastic-plastic strip has been calculated from elementary bending theory. Measured forces for Mylar strips of various thicknesses, bent to various degrees, were in good agreement with these calculated values. The corresponding additional stripping force in a peeling experiment will depend upon the thickness of the elastic-plastic adherend, becoming zero both for infinitesimally thin adherends and for those exceeding a critical thickness  $t_c$  and passing through a maximum value at intermediate thicknesses. Published data are in good agreement with these conclusions. For a strongly adhering strip, higher peel strengths are found for a peel angle of 180°, compared to 90°, and the effect is greater than can be accounted for solely by plastic yielding of the adherend. It is attributed in part to greater energy dissipation within the adhesive layer.

## INTRODUCTION

Several investigators have analyzed the peeling test assuming that both the adherend and the adhesive obey linear elastic mechanics.<sup>1-7</sup> However, it is now widely recognized that in a peeling experiment the stripping member may undergo plastic yielding if the bending stresses imposed by the peel force are sufficiently large.<sup>8–13</sup> Plastic yielding provides an energy dissipation mechanism, and thus a higher peel force is required than if yielding does not occur. The magnitude of this additional energy dissipation has now been determined experimentally for a simple elastic-plastic strip adhering to a rigid substrate. Results are given below for the peel force component arising from yielding of the adherend. They are compared with the predictions of an approximate theoretical treatment. Conclusions are drawn as to the effect of yield stress, thickness, and elastic modulus of the adherend, and of the strength of adhesion to the substrate.

In a further series of experiments, the contribution to the work of detachment arising from plastic yielding of a strip adhering to a deformable elastomeric substrate has been measured. Peeling experiments have been carried out at peel angles of 90° and 180°; the results are compared with values obtained when plastic yielding of the strip was prevented. Large differences were observed for both peel angles. They are attributed only in part to energy expended in plastic deformation of the adherend; part of the additional energy losses must arise within the elastomeric substrate when plastic yielding occurs in the detaching layer. Dissipative processes within the adhesive layer have previously been shown to account for much of the observed peel strength of adhesive joints, in the absence of plastic yielding.<sup>14-17</sup> These processes are not discussed in the

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present paper, which deals solely with the conditions for, and consequences of, plastic yielding in the detaching layer.

# EXPERIMENTAL

As shown later, Mylar [poly(ethylene terephthalate) film, E. I. du Pont de Nemours and Co.] undergoes plastic yielding at a well-defined yield stress and conforms closely to ideal elastic-plastic behavior, at least in tension. Strips of Mylar were, therefore, used as model elastic-plastic adherends. The force P per unit width of strip required to propagate a bend was measured for varying degrees of curvature of the strip during detachment from a weakly adhering substrate. As shown in Figure 1, two Mylar strips were detached simultaneously by peeling them away from two rubber-covered metal plates. Adhesion to the rubber was relatively small, the peel force P being only about 4 N/m in the absence of plastic yielding. The thickness of the rubber layer was also small, about 0.3 mm, so that no significant deformation of the rubber was expected. The rubber layers served merely to prevent slipping and buckling of the Mylar strips under the force P.

The degree of bending of the Mylar strips was characterized by the distance D between the central planes of the adhering and fully removed portions of the film, Figure 1. Various values of D were obtained by altering the separation of the two steel backing plates, which were held parallel to each other at a given distance by means of adjustable spacer rods. When the spacing D was reduced below a critical value, the force P was found to increase greatly, and the amount, denoted  $P_y$ , by which it exceeded the small value required to detach the unrestrained strip is attributed to energy dissipated in plastic yielding. Values of  $P_y$  were measured for a wide range of spacings D, and for various thicknesses of film, in the range of 25–360  $\mu$ m. In addition, the effect of varying the speed of propagation of the bend was examined over the range of  $3 \times 10^{-4}$  mm/sec to 3 mm/sec.

In order to examine the effect of plastic yielding under conditions of strong adhesion, Mylar strips 76  $\mu$ m thick were adhered to a 1 mm thick layer of an elastomeric SBS triblock copolymer (Shell Kraton 1101) by pressing the two materials together for 90 min at a temperature of 150°C. The Kraton layer was held flat by bonding its lower surface to a steel plate. On peeling the Mylar strip off the elastomer, failure occurred at the Mylar–elastomer interface, as represented schematically in Figure 2, and the detached strip was tightly curled, indicating that it had undergone severe plastic yielding during detachment.

Peeling experiments were also carried out with the same strongly adhering materials, i.e., Mylar and Kraton 1101, under conditions where no plastic deformation of the Mylar strip occurred. The experimental arrangement for this, suggested by Dr. D. I. Livingston of the Goodyear Tire and Rubber Company Research Division, is shown in Figure 3. It resembles that employed in ASTM Test Method D 3167-73T. The Mylar strip in the detachment region was bent around a freely rotating roller having a sufficiently large radius of curvature so that bending stresses in the Mylar would not cause yielding. With a 12.7-mm-diameter roller, the maximum tensile strain in the outer regions of the Mylar strip was calculated to be only 0.006, considerably smaller than the yield strain, as discussed later.

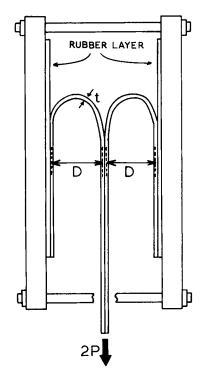


Fig. 1. Experimental arrangement for determining the work expended in plastic deformation of a peeling strip.

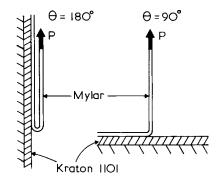


Fig. 2. Peeling an adhering Mylar strip from a Kraton 1101 substrate at 180° and 90°.

Weights were applied to the roller axle, as shown in Figure 3, so that the Mylar film would conform closely to the surface of the roller as detachment occurred. It was also found necessary to pull at a slight angle to the vertical in order to assure that the Mylar film was in good contact with the roller. Once the weights were in place and before starting to peel, the force measured by the load cell due to the weights and roller arrangement was noted. This value was then subtracted from the force measured during detachment to give the true peel force. When removed in this way, the Mylar strip showed no residual curvature, indicating that no plastic yielding had occurred during detachment.

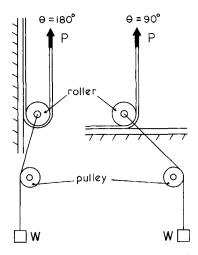


Fig. 3. Experimental arrangement for peeling at 180° and 90° without plastic yielding of the detaching strip.

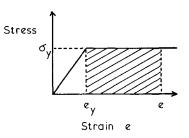


Fig. 4. Stress-strain relation for an ideal elastic-plastic solid.

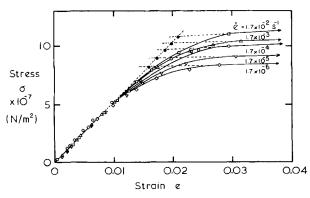


Fig. 5. Tensile stress-strain relations for Mylar film stretched at various rates  $\dot{e}$ .

# THEORETICAL CONSIDERATIONS: PEELING AN ELASTIC-PLASTIC ADHEREND

An ideal elastic-plastic solid follows a linear stress-strain relation until the yield stress  $\sigma_y$  and yield strain  $e_y$  are attained. It then deforms at constant stress, as shown schematically in Figure 4. Mylar film shows elastic-plastic behavior in tension resembling this ideal pattern, (Fig. 5), although the transition from

é, sec <sup>-1</sup>	$\sigma_{y,}$ MN/m <sup>2</sup>	ey
$1.7 \times 10^{-6}$	82	0.016
$1.7 \times 10^{-5}$	89	0.0175
$1.7  imes 10^{-4}$	95	0.0185
$1.7 \times 10^{-3}$	101	0.0195
$1.7 \times 10^{-2}$	107	0.021

TABLE I Values of Yield Stress  $\sigma_{y}$  and Yield Strain  $e_{y}$  for Mylar Film Stretched at Various Rates

elastic to plastic behavior is more gradual than in the ideal case. Moreover, the yield stress and yield strain increase somewhat with rate of deformation, even though the elastic modulus appears to remain substantially unchanged (Fig. 5).

Approximate values for yield stress and strain were obtained from the intercept of the two linear relations which describe the wholly elastic and wholly plastic regimes, represented by the broken lines in Figure 5. The filled-in circles in Figure 5 denote yield points deduced in this way; the numerical values are given in Table I and are plotted in Figure 6 against the rate of extension  $\dot{e}$  on a logarithmic scale. As is commonly found,<sup>18</sup> these semilogarithmic plots yielded linear relationships. When values of yield stress and strain were required in order to calculate the plastic work expended in peeling, using the approximate theory developed below, the corresponding rate of strain was approximated by

$$\dot{e} = \dot{c}/t$$

where  $\dot{c}$  is the rate of peel and t is the thickness of the strip. Values of  $\sigma_y$  and  $e_y$  were then read from Figure 6.

A schematic diagram of one of the peeling Mylar strips is shown in Figure 7, with the distribution of stress across the thickness shown for a particular section which has undergone partial plastic yielding. It is assumed that the yield behavior in compression is the same as that in tension, so that the neutral axis is still located at the center of the strip. As a section of the strip traverses the bend, it passes from an undeformed state in the adhering region through a maximum

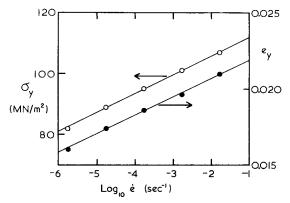


Fig. 6. Experimental relations between yield stress  $\sigma_y$ , yield strain  $e_y$ , and rate of extension  $\dot{e}$ , from Fig. 5.

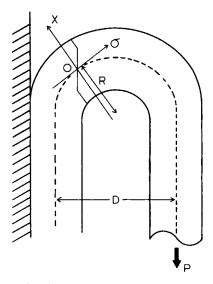


Fig. 7. Schematic diagram of peeling strip showing neutral axis (broken curve), local radius of curvature R, and the variation of tensile stress  $\sigma$  with distance x from the neutral axis.

bending deformation where the radius of curvature of the neutral axis is a minimum, at the point O in Figure 7, say. The maximum strain e imposed on a layer at a distance x from the neutral axis is then given by

$$e = x/R \tag{1}$$

where R is the minimum radius of curvature of the neutral axis. If this strain exceeds the yield strain  $e_y$ , then energy is dissipated per unit volume equal to the shaded area in Figure 4 in taking the layer past the point O. If e is less than  $e_y$ , then it is assumed that no energy is expended in taking the layer around the bend, i.e., all the elastic energy expended in deforming the layer is recovered again when it is straightened. Also, any further plastic work expended as the plastically deformed layers move on into regions of lesser curvature and are straightened again is neglected. In practice, the peeled strips were found to show residual curvature after plastic deformations had been encountered, suggesting that any plastic work expended subsequently in straightening them was less than that expended in deforming them.

Thus, the total energy expended in plastic deformation during peeling of unit length of the strip is given approximately by

$$W = 2 \int_{e_y R}^{t/2} \sigma_y (e - e_y) dx \tag{2}$$

per unit width of strip, where t is the thickness. This energy is supplied by the component  $P_{y}$  of the peel force arising from plastic work:

$$W = 2P_{\gamma} \tag{3}$$

Hence, from eq. (1), (2), and (3),

$$P_{y} = \frac{1}{4} \left( \sigma_{y} e_{y} t \right) \left[ \left( \frac{t}{2Re_{y}} \right) + \left( \frac{2Re_{y}}{t} \right) - 2 \right]$$
(4)

It is now necessary to find the relationship between the minimum radius of curvature R and the imposed spacing D in order to compute the corresponding peel forces  $P_y$  by means of eq. (4). Two limiting cases will be considered. For an elastic strip peeled at 180°, the distance D is obtained from elementary bending theory<sup>19</sup>:

$$D = 2(EI/P)^{1/2}$$
(5)

where E is Young's modulus and I is the second moment of area of cross section per unit width of the strip, given by

$$I = t^{3}/12$$

The minimum radius of curvature in this case is developed at the point of separation; it is

$$R = EI/PD \tag{6}$$

Combining eqs. (5) and (6), we obtain a relation for the minimum radius of curvature in terms of the spacing D for a perfectly elastic strip,

$$R = D/4 \tag{7}$$

The other limiting case occurs when the peeling strip folds back on itself to form a fully developed plastic hinge at the point of detachment. The minimum possible value of the radius of curvature of the neutral axis in this case is t/2, when the separation D of the neutral axes becomes equal to the strip thickness t. Thus,

$$R \approx t/2 \approx D/2 \tag{8}$$

In practice, the minimum radius R will lie between the extremes given by eqs. (7) and (8), tending toward the former relation when the amount of plastic deformation is small and toward the latter, when it is substantial.

It is instructive to examine eq. (4) in greater detail. At small values of the spacing D and radius of curvature R, the first term in the brackets in eq. (4) becomes dominant and the equation simplifies to

$$P_{\gamma} \approx \sigma_{\gamma} t^2 / 8R \tag{9}$$

When the radius of curvature takes its minimum possible value, t/2, the corresponding maximum possible contribution to the peel force per unit width is obtained as

$$P_{y,\max} = \sigma_y t/4 \tag{10}$$

It should be noted that the peeling strip will fail by plastic yielding in tension at a force of  $\sigma_y t$  per unit width. The maximum peel force contribution from plastic bending effects is thus comparable in magnitude to the ultimate strength of the adherend.

At the other extreme, eq. (4) predicts that  $P_y$  becomes zero when the radius of curvature exceeds a critical value, denoted  $R_0$ , given by

$$R_0 = t/2e_y$$

The corresponding critical spacing  $D_0$  for elastic films is given by eq. (7):

$$D_0 = 2t/e_y \tag{11}$$

and the corresponding critical thickness  $t_c$  is given by

$$t_c = 12 \, E P_0 / \sigma_v^2 \tag{12}$$

from eqs. (5), (6), and (11), where  $P_0$  denotes the peel force per unit width in the absence of plastic yielding. No contribution to the observed peel force from plastic yielding will occur for adhering strips having a thickness greater than  $t_c$ .

Calculations up to this point have dealt with peeling at an angle of 180°. It is instructive to consider what changes would be necessary for peeling at 90°. Equation (9) becomes

$$P_{\nu} \approx \sigma_{\nu} t^2 / 4R$$

for the peel force contribution at small values of the radius of curvature R. When the radius takes its minimum feasible value, t/2, the corresponding maximum possible value of  $P_{\nu}$  becomes

$$P_{y,\max} = \sigma_y t/2$$

in place of eq. (10), even closer than before to the ultimate strength of the peeling strip. When the thickness of the peeling strip exceeds a critical value, given by

$$t_c = 6EP_0/\sigma_v^2 \tag{13}$$

plastic yielding will not occur. At first sight, eq. (13) appears to predict a smaller value of critical thickness for an adherend peeled at 90° than for one peeled at 180°, eq. (12). However, because the peel force at 90° is twice as large as that at 180° for the same work of detachment, eq. (12) and (13) actually correspond to the same value of critical thickness.

# EXPERIMENTAL RESULTS FOR A WEAKLY BONDED ELASTIC-PLASTIC ADHEREND

In Figure 8, the total peel force P per unit width of strip is plotted as a function of peel rate for a Mylar film weakly adhered to a rubber-coated steel plate and held at various degrees of bending during detachment, Figure 1. The peel forces for an unrestrained strip (where the value of the spacing distance D is approximately 15 mm) are also shown in Figure 8. As D was reduced down to about 4 mm, the peel forces remained small and constant. But for values of D below this critical level, i.e., for higher degrees of bending of the Mylar strip, a dramatic increase in P was found.

The additional peel force  $P_y$  was measured for various values of the imposed spacing D below the critical value (denoted  $D_0$ ). The results are shown in Figure 9 for four different thicknesses of Mylar film at a constant peeling speed of 0.04 mm/sec. For a fixed spacing,  $P_y$  was found to be greater the thicker the Mylar film. Also, the values obtained dropped rapidly to zero as the spacing approached the corresponding critical value  $D_0$  for each film.

In order to obtain theoretical values of  $P_y$  for quantitative comparison with these experimental results, several steps are necessary. First, values of yield stress  $\sigma_y$  and strain  $e_y$  were read from Figure 6 at the appropriate rate of extension. The minimum radius R of curvature appropriate to the spacing D was

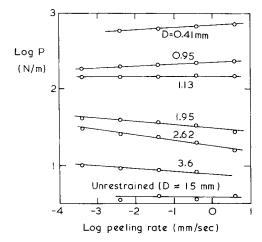


Fig. 8. Peel force P per unit width vs rate of peeling for a Mylar film 76  $\mu$ m thick, peeled at various degrees of bending. D denotes the imposed bending spacing, as shown in Fig. 1.

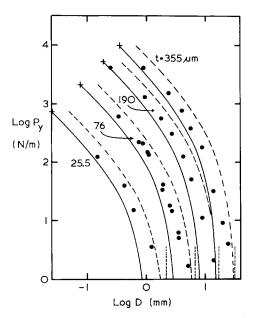


Fig. 9. Additional peel force  $P_y$  per unit width arising from plastic yielding vs imposed bending spacing D, for Mylar films of various thickness t peeled at 0.04 mm/sec. Vertical broken lines denote calculated values of the critical spacing  $D_0$ , from eq. (11).

determined from eq. (7) or (8). Values of  $P_y$  were then calculated from eq. (4). The two theoretical relations shown in Figure 9 for each thickness of Mylar film were obtained in this way. The broken curves, using R from eq. (7), should be more appropriate at small amounts of plastic work and the full curves, using R from eq. (8), at large degrees of plasticity.

Maximum values of  $P_y$  were also calculated from eq. (10). They are plotted in Figure 9 (crosses) against the appropriate values of D for each film, i.e., when D = t, and form the terminal points for the theoretical relations representing large degrees of plasticity, the full curves in Figure 9. Values of the spacing  $D_0$  at which the contribution  $P_y$  to the peel force becomes zero were calculated from eq. (11). They are shown in Figure 9 as the right-hand vertical asymptotes of the relations representing small amounts of plastic work, i.e., the broken curves in Figure 9. Experimentally observed spacings at which a plastic contribution to the peel force was first observed were in good agreement with these calculated values in all cases.

Indeed, the experimental results for  $P_y$  are seen to be in good agreement with the theoretical predictions over the entire range of measured values and for all thicknesses of adherend. Moreover, they tend to lie closer to the broken relations at large spacings D and closer to the full relations at small values of D, in accord with expectation. Contributions to the observed peel forces arising from plastic work are thus successfully accounted for, both qualitatively and quantitatively, by the present theoretical treatment.

Effects of rate of peeling on the observed peel force can also be explained in terms of contributions from plastic work. At values of the spacing distance Djust below the critical level  $D_0$ , the peel force was found to decrease as the peel rate increased, whereas at much smaller values of D than this, the peel force increased somewhat with increasing peel rate, Figure 8. Both of these contrasting effects appear to be direct consequences of the rate dependence of yield stress and yield strain, Figure 6. At large values of D, i.e., at strains near the onset of plasticity, yielding will be more extensive at low rates because the yield strain is smaller at low rates of deformation. The contribution  $P_y$  to the observed peel force will be correspondingly greater at low rates of peel and decrease as the rate increases. However, when plastic yielding is already extensive, i.e., at small values of the spacing D, then the peel force will increase with increasing rate of peel because of the corresponding increase in yield stress and strain with rate of deformation.

### EXPERIMENTAL RESULTS FOR A STRONGLY ADHERING STRIP

Measurements were made of the work of detachment for a Mylar strip 76  $\mu$ m thick adhering to a layer of Kraton 1101. For peeling at 90°, the work of detachment is given directly by the peel force per unit width, but for peeling at 180°, it is given by twice the peel force per unit width because, in order to detach a length L, the point of application of the peel force must travel a distance 2L.

The experimental results at both peel angles are plotted in Figure 10 against the rate of peeling. As shown there, the work of detachment was found to be considerably larger at a peel angle of 180°, compared to 90°. Moreover, after detachment at 180°, the Mylar strip exhibited a high degree of permanent curvature, indicating that extensive plastic yielding had occurred during the peeling process, whereas after peeling at 90°, the residual curvature was much less pronounced.

The theoretical treatment given earlier does not predict any difference between the contribution of plastic yielding to the observed work of detachment at 90° and 180°. The present observations cannot, therefore, be accounted for in terms of that theory, and an explanation must be sought in other directions.

It is illuminating in this connection to compare the observed works of detachment with that required in the absence of plastic yielding. Detachment of the Mylar strip at 90° and 180° without plastic deformation was achieve by

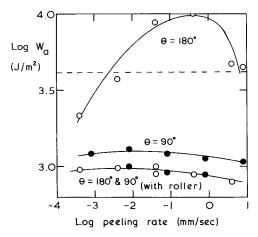


Fig. 10. Work  $W_a$  of detachment vs. rate of peeling for a Mylar strip 76  $\mu$ m thick adhering to a Kraton 1101 substrate: (O) peeled at 180°; ( $\bullet$ ) peeled at 90°. Broken horizontal line represents the maximum energy expended in plastic deformation of the Mylar strip at 180°, calculated from eq. (10).

peeling around a large-diameter roller, as shown in Figure 3. The results obtained in this way are included in Figure 10. They were much smaller than before, and the values for 90° and 180° peel angles now coincided. Also, no permanent curvature of the Mylar strips was found. Thus, the difference found previously between the work of detachment at 90° and 180° peel angles can be attributed, at least in part, to different levels of plastic work in the two cases, although it is not yet apparent why they should differ.

The maximum contribution that yielding in the Mylar strip can make to the detachment energy may be calculated by means of eq. (10). The value obtained is represented by the broken horizontal line in Figure 10. When it is added to the lower curve in Figure 10, representing the detachment energy when no vielding of the Mylar strip occurs, the sum is still not as large as the highest values obtained experimentally for peeling at 180°. This discrepancy is even greater when the actual contribution to the detachment energy arising from plastic yielding of the Mylar strip is employed in place of the maximum possible value. The actual work expended in plastic yielding was deduced from measurements of the bending distance D, obtained from photographs of the peeling strip in the process of detachment. A value for D of 0.32 mm was determined in this way for peeling at 180° under the largest peel forces. The corresponding contribution of plastic yielding to the work  $W_a$  of detachment was obtained from Figure 9; it is only about  $1.3 \times 10^3$  j/m<sup>2</sup>. This is far too small to account for the difference between the value of  $W_a$  when no plastic yielding occurs, about  $1 \times 10^3$  j/m<sup>2</sup>, and the value obtained when the Mylar strip is allowed to bend sharply during detachment, about  $1 \times 10^4$  j/m<sup>2</sup> (Fig. 10).

There are, therefore, two anomalous features of the present experimental results: the Mylar strip does not undergo fully plastic bending, even though the peel forces are sufficiently large to cause this condition, and the peel energy at 180° is considerably greater than the sum of the plastic work expended in the Mylar and the work of detachment in the absence of plastic yielding. These discrepancies appear to be associated with deformations of the adhesive layer

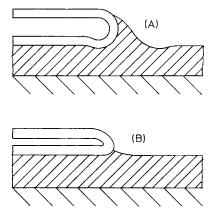


Fig. 11. Detachment from a deformable substrate (a) and from a stiffer, less extensible substrate (b).

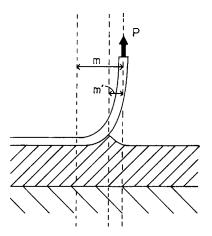


Fig. 12. Reduction in bending moment for peeling at 90° from a deformable substrate (schematic). The moment arm of the peel force P is represented by m for a rigid substrate and by  $m^1$  for the deformable substrate shown.

in the immediate vicinity of the detachment front. Part of the adhesive material was observed to follow around the sharply bent region of the Mylar strip before detaching, as shown schematically in Figure 11(a). Thus, the Mylar strip was stiffened by this still-adhering part of the substrate layer, and it did not undergo such severe bending as it otherwise would have done [Fig. 11(b)]. This is apparently the reason why it did not become fully plastic, even though the peel forces were large enough to make an unsupported Mylar film yield completely.

Deformation of the substrate layer will be still more effective in lessening the effects of plastic yielding at a peel angle of 90°. In this case, a relatively small deformation of the substrate will allow a significant *rotation* of the still-attached portion of the adherend toward the line of action of the peel force. Thus, the bending moment acting at the point of detachment will be decreased and yielding of the adherend will be delayed or reduced (Fig. 12). We conclude that strongly-adhering substrates which also have sufficient extensibility to conform to the shape of the adherend in the region of detachment will reduce the severity

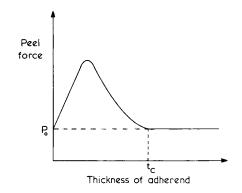


Fig. 13. Dependence of the peel force P on thickness t of an elastic-plastic adherend. Broken horizontal line represents the work of detachment in the absence of plastic yielding.

of bending and hence reduce the contribution of plastic yielding of the adherend to the work of detachment, especially at 90°.

The question remains, why are the observed energies of detachment for peeling at 180° so large? An important contribution appears to come from energy dissipation within the substrate material in the highly deformed region around the point of detachment. The present substrate, although basically elastomeric in character, is stiffer for small deformations and then yields at a tensile stress of about 1.5 MN/m<sup>2</sup>, to become softer and highly extensible. Thus, additional energy dissipation by a yielding process within the substrate will occur when the local stress reaches a value of about 1.5 MN/m<sup>2</sup>. This circumstance may well arise in the vicinity of a sharply bent peeling strip. Moreover, dissipation of energy by this mechanism is likely to depend strongly upon the rate of deformation because the substrate material shows rate-dependent mechanical properties. Thus, the pronounced effect of the rate of peel upon the work of detachment at 180° (Fig. 10), is consistent with rate effects within the substrate material rather than within the Mylar adherend.

We conclude that inelastic deformation of the substrate layer was mainly responsible for the higher peel strengths observed at a peel angle of 180°. Plastic deformation of the stripping member can thus lead to an increase in peel strength in two ways: (i) directly, by an additional force representing the work required to propagate the bend in an elastic-plastic strip as peeling proceeds, and (ii) indirectly, by causing a larger deformation in the elastomeric substrate under the higher peel forces and bringing about greater energy losses in this layer as a result.

# EFFECT OF THICKNESS OF THE ADHEREND

It is apparent from Figures 9 and 10 that the energy expended in bending the stripping layer may make a large contribution to the total work of detachment. The extent of this contribution will depend upon the strength of adhesion, however. If the interfacial adhesion is sufficiently weak or if the adhering layer is sufficiently thick or strong, the conditions for plastic yielding will not arise and there will be no contribution to the peel force from this source. On the other hand, with strong adhesion, or thin ductile adhering layers, yielding will occur readily and contribute to the total work of detachment.

These considerations account for published reports<sup>20–22</sup> that the peel force passes through a maximum as the adherend thickness is increased. For a given level of adhesion, a very thin adherend will yield during peeling, but the total energy dissipated will be small because t is small, and the contribution of plastic yielding to the peel force will be negligible. As the thickness is increased, more energy will be dissipated and the peel force will increase, eq. (10). However, at sufficiently large thicknesses, the detaching layer will become too stiff to undergo yielding throughout its thickness and the peel force will begin to decrease as the thickness is increased further. Eventually, when the thickness exceeds  $t_c$ , the detaching layer will not undergo plastic yielding at all and the peel force will return to its original value. Thus, the peel force at both zero and large thicknesses of adherend should be equal, and they should both reflect the work of detachment in the absence of plastic yielding. This general dependence upon thickness is shown schematically in Figure 13; published results are in good agreement with these deductions.<sup>20,21</sup>

### CONCLUSIONS

The peel force required to propagate a bend in an elastic-plastic strip has been calculated using elementary bending theory. Measured forces for Mylar strips of various thicknesses, bent to various degrees, have been found to be in good agreement with these calculated values. The extent to which these forces contribute to the total stripping force in a peeling experiment is governed by the strength of adhesion, relative to the thickness and yield stress of the adherend. While energy expended in the stripping member can clearly make a major contribution to the total peel force (Fig. 9 and 10), the contribution will be zero when the interfacial adhesion is relatively small and the adherend is sufficiently thick so that it does not undergo plastic yielding at all.

The deformability of the adhesive will also affect the extent to which plastic yielding contributes to the total peel force. For instance, consider the two peel systems shown in Figure 11, which both peel under the same force P. Case A shows peeling with a soft, deformable adhesive, whereas in case B, the adhesive is nearly rigid. In case A, the detaching strip may be kept from undergoing a sharp bend because the adhesive effectively stiffens it in the peeling region. Thus, the contribution of yielding to the total peel force will be reduced in this case. In case B, the same peel force may cause the detaching strip to pass through a sharp bend, with a consequent large contribution to the total peel force arising from plastic yielding.

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